UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2007 question paper

4037 ADDITIONAL MATHEMATICS

4037/01   Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M   Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B   Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
                B2, 1, 0 means that the candidate can earn anything from 0 to 2.
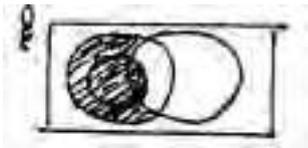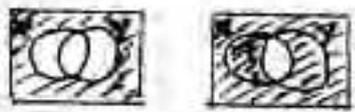
The following abbreviations may be used in a mark scheme or used on the scripts:

AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD   Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO   Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW   Ignore Subsequent Working

MR   Misread

PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS   See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR -1   A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW -1,2   This is deducted from A or B marks when essential working is omitted.

PA -1   This is deducted from A or B marks in the case of premature approximation.

S -1   Occasionally used for persistent slackness – usually discussed at a meeting.

EX -1   Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1. (i)	B1 [1]	co
(ii) $A \cap B' \cap C$	B1 [1]	co
(iii) $(X \cup Y)'$ $X' \cup Y'$	B1 B1 [2]	co co.

2. $y = \dfrac{2x+4}{x-2}$ $dy/dx = \dfrac{(x-2)2 - (2x+4)}{(x-2)^2}$ If $x = 4$, $dy/dx = -2$ Perpendicular has $m = \frac{1}{2}$ If $x = 4$, $y = 6$ $\rightarrow$ Eqn $y - 6 = \frac{1}{2}(x - 4)$ $[2y = x + 8]$	M1 A1 M1 B1 A1 [5]	Formula must be completely correct co. (may be implied) Independent of first M mark. Anywhere in the question.

3. $3x = 2y + 18$ $2x^2 - 23x + 2y + 50 = 0$ $\rightarrow$ $x^2 - 10x + 16 = 0$ or $y^2 + 3y - 18 = 0$ $\rightarrow \rightarrow$ $(2, -6)$ and $(8, 3)$ Vector moves or other $\rightarrow$ $P(4, -3)$	M1A1 DM1 A1 M1A1√ [6]	Complete elimination of x/y for M. Correct method of solution of quad. Any valid method.

4. (i) $(2 + u)^5 = 32 + 80u + 80u^2$
 	
 (ii) Replaces u by $2x - 5x^2$
    -400 from 'u' term or $+320$ from 'u²' term
    Also … $+80(2x - 5x^2)^2$
    $\rightarrow -400 + 320 = -80$
	B2,1,0
	 [2]
	 M1
	 B1
	 M1
	 A1√
	 [4]
		One lost for each error
		
		 Recognises and uses the link.
		 Co (may be implied by answer)
		 Needs to look at 2 terms for x^2
		 From his original expansion.

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5. $y = \sqrt{x} + \dfrac{9}{\sqrt{x}}$

(i) $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}} - \dfrac{9}{2x^{\frac{3}{2}}}$ B1 B1

$\dfrac{d^2 y}{dx^2} = \dfrac{-1}{4x^{\frac{3}{2}}} + \dfrac{27}{4x^{\frac{5}{2}}}$ B1 B1

[4]

Accept all these B marks if given as negative powers of x

(ii) If $x = 9$, $\dfrac{dy}{dx} = 0$ B1   [1]   Answer given.

.

(iii) If $x = 9$, $\dfrac{d^2 y}{dx^2} > 0$. Minimum   M1 A1   [2]

Looks at sign of $\dfrac{d^2 y}{dx^2}$. Needs all correct for the A mark.

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6. (i) In 1.8s, alien goes 27 cm up.   B1
     In 1.3 s missile goes 39 up.   B1

     But alien starts at 12 up.
     → $39 - 27 = 12$ M1 A1   [4]   Equates 2 vertical displacements.

   (ii) In 1.8s. alien goes 72 across
      In 1.3 s, missile goes $1.3k$ B1

      $72 = 1.3k + 46$ → $k = 20.$ M1 A1   [3]   Equates 2 horizontal displacements.

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7. (a) $5^{x+1} = 8 + 4(5^{-x})$ → $5u = 8 + 4u^{-1}$ B1 B1   B1 for $5u$ and B1 for $4u^{-1}$

     → $5u^2 - 8u - 4 = 0$
     → $u = 2$ or -0.4 M1   Solution of a quadratic.
     Soln of $5^x = 2$ → $x = \lg 2 \div \lg 5$ M1   Allow for any soln of $5^x = k$.
     → $x = 0.431$ A1   [5]   co.

   (b) $\log(p - q) = \log p - \log q$

         $= \log(p/q)$ B1   co.
     $p - q = p/q$ M1   Eliminating lg + good algebra.

     → $p = \dfrac{q^2}{q-1}$ A1   [3]   co.

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8. (a) $1 + 5\cos 3x = 0$
     $\cos 3x = -0.2$ $3x = \cos^{-1}(-0.2)$ M1   Looks up cos before ÷ 3
     → $x = 0.59$ or 1.50 A1 A1   [3]   co.co.

   (b) $\sec y + 5\tan y = 3\cos y.$

     $\sec y = 1/\cos y$ and $\times \cos y$ M1   Needs both of these.
     uses $\cos^2 = 1 - \sin^2$ M1   Needs correct link.
     → $3\sin^2 y + 5\sin y - 2 = 0$ + solution   DM1   Solution of quadratic
     → $\sin y = \frac{1}{3}$ $y = 19.5°$ and $160.5°$.   A1 A1√   [5]   co. √ for 180 − (first ans)

9. (i)

$1/x$	10	8	6.25	5	2.5
$1/y$	20	15.6	11.8	9.0	3.5

M1   Knows what to do.
A2,1,0 Accuracy.
    [3]

(ii) Gradient 2.2 (±0.05) Intercept = $-2(\pm0.1)$

$$\frac{1}{y} = 2.2.\frac{1}{x} - 2$$

$$\rightarrow \quad y = \frac{x}{2.2 - 2x}$$

B1 B1   Within given range – graph needed

M1   Uses $Y = mX + c$

A1√   Correct form with his m and c.
  [4]

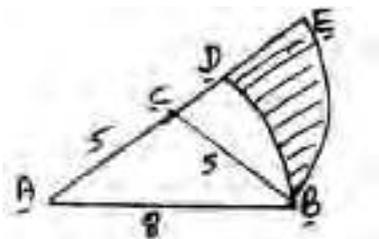(iii) $y = 0.15$ $1/y = 6.7 \rightarrow 1/x = 4$

$$\rightarrow x = 0.254 \ (\pm0.010)$$

M1   Uses $1/y$ and $1/x$ correctly – or solves equation from part (ii).
A1   co within range.
  [2]

10

(i) $AC = \cos^{-1}4/5 = 0.6435$ rads
    $BCE = 2 \times BAC = 1.287$

M1   Complete method inc radian use.
A1   co – answer given.
  [2]

(ii) arc $BD = 8 \times 0.6435 = (5.148)$
    arc $BE = 5 \times 1.287 = (6.435)$
    $DE = 10 - 8$

$\rightarrow$ Perimeter = sum of these = 13.6 m.

M1   Any use of $s = r\theta$

B1   Anywhere

DM1 A1 Sum of three parts. co.
  [4]

(iii) Area of Δ $ABC = 3 \times 4$ or $\frac{1}{2}ab\sin C = 12$
  Area of sect $CBE = \frac{1}{2} \times 25 \times 1.287 = (16.09)$
  Area of sect $ABD = \frac{1}{2} \times 64 \times 0.6435 = (20.59)$

$\rightarrow$ shaded area = $12 + 16.09 - 20.59$
$\rightarrow$ 7.50 m^2

M1   Correct method for triangle.
M1   Any use of $A = \frac{1}{2}r^2\theta$

M1   Must be linked correctly. Not DM.
A1   Correct to 3 sf.
  [4]

Q 9

$c. -2.0$

$m = \dfrac{+22}{10} = +2.2$

$\frac{1}{y} = 6.7$
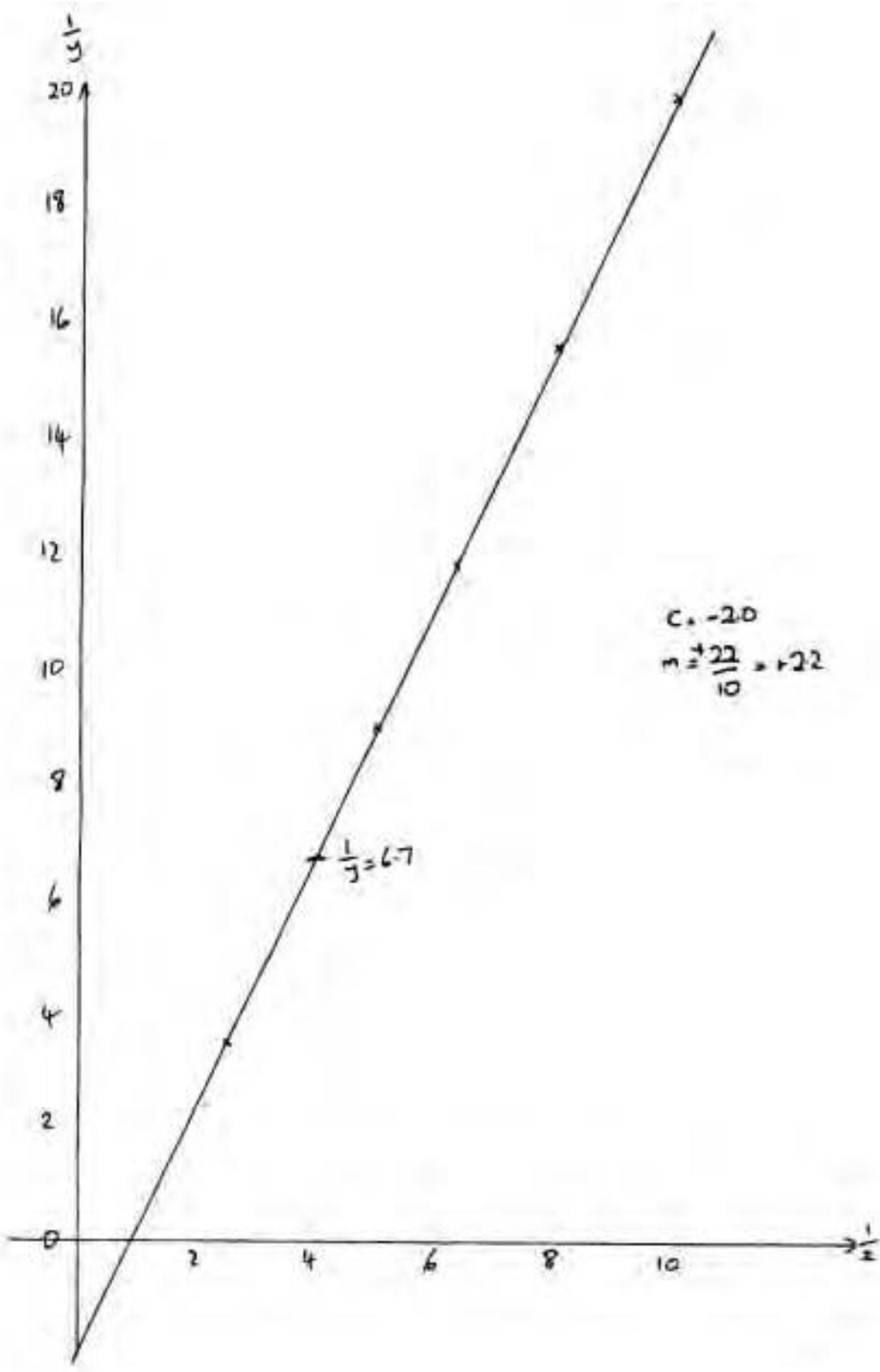
11 EITHER
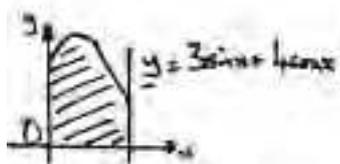
(i) $dy/dx = 3\cos x − 4\sin x$ M1 A1   Attempt at differentiation. co.

     $= 0$ when $3\cos x − 4\sin x = 0$ DM1   Sets differential to 0.

     $\tan x = ¾$ DM1   Arrives at $\tan\theta = k$.

     $\rightarrow x = 0.644$ A1   Both x and y needed.

     $\rightarrow y = 5.00$ [5]

(ii) $A = \int_{0}^{\frac{\pi}{2}} 3\sin x + 4\cos x\,.dx$ M1   Any attempt to integrate

     $= [−3\cos x + 4\sin x]$ A1 A1   Each term.

     $= [0+4\} − [−3+0]$ DM1   Correct use of limits – DM0 if "0" left
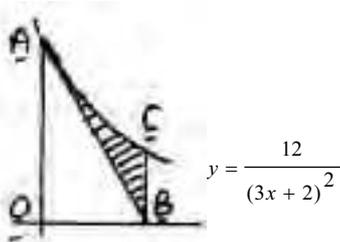
     $\rightarrow 7$ A1   co   [5]

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11 OR

$y = \dfrac{12}{(3x+2)^2}$

(i) $dy/dx = −24 \times (3x+2)^{−3} \times 3$ B1 B1   For $−24 \times (3x+2)^{−3}$, for $\times 3$

     When $x = 0$, $dy/dx = −9$ B1   co.

     At A, $x = 0$ and $y = 3$

     $\rightarrow B: x = ⅓$ B1   co   [4]

(ii) $A = \int_{0}^{1/3} \dfrac{12}{(3x+2)^2}\,.dx$ M1   Attempt needed to integrate

     $= [−12(3x+2)^{−1} \div 3]$ A1 A1   For $−12(3x+2)^{−1}$.   For $\div 3$.

     $= −4/3 − −2 = ⅔$ DM1   Not given if bottom limit ignored.

     Area of triangle $= ½ \times 3 \times ⅓ = ½$ M1   Anywhere.

     $\rightarrow A = 1/6$ A1   co   [6]

DM1 for quadratic equation. Equation must be set to 0 if using formula or factors.

<u>Formula</u>.   <u>Factors</u>

Must be correct   Must attempt to put quadratic into 2 factors.

– ignore arithmetic and algebraic slips.   Each factor then equated to 0.